

Soltaniyeh and Santa Maria del Fiore: A historic relationship?

Soltaniyeh y Santa Maria del Fiore: ¿una relación histórica?

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Abstract

As one of the earliest human innovations in the architecture of shells, the dome has undergone many changes. This article examines the theory of the Florence Cathedral's (Santa Maria del Fiore) inspiration from the Soltaniyeh Dome of Zanjan. The methodology employed in this research is comparative-descriptive using reliable historical and library sources. A comparative-historical study of 13 domes in Iran (Iranian plateau) and 13 domes in Italy (Eastern Rome Empire) over 2,000 years has examined the evolution of Iranian and Italian domes in terms of construction time and shape. In the next step, the sources that architectural experts have discussed the influence of the dome of Soltaniyeh in Zanjan on the dome of Santa Maria del Fiore have been reviewed. The influence of the golden age of Islam and the achievements of Muslim scholars on the European renaissance in mathematics and geometry has also been explored. The structural similarities, proportions and forms of these two domes show impressive results. It seems that according to historical sources and comparative studies conducted in this study, the dome of Soltaniyeh has a direct impact on the construction of the dome of Santa Maria del Fiore, and the theory first proposed by Professor Sanpaolesi (1972) is conclusive.

Keywords: Soltaniyeh, Santa Maria del Fiore, Dome, Islamic Golden Age, Renaissance.

Resumen

Como una de las primeras innovaciones humanas en la arquitectura de las conchas, la cúpula ha sufrido muchos cambios. Este artículo examina la teoría de la inspiración de la Catedral de Florencia (Santa Maria del Fiore) de la Cúpula Soltaniyeh de Zanja. La metodología empleada en esta investigación es comparativa-descriptiva utilizando fuentes históricas y bibliográficas fidedignas. Un estudio histórico comparativo de 13 cúpulas en Irán (meseta iraní) y 13 cúpulas en Italia (Imperio de Roma Oriental) durante 2000 años ha examinado la evolución de las cúpulas iraníes e italianas en términos de tiempo de construcción y forma. En el siguiente paso, se han revisado las fuentes en las que los expertos en arquitectura han discutido la influencia de la cúpula de Soltaniyeh en Zanja en la cúpula de Santa Maria del Fiore. También se ha explorado la influencia de la edad de oro del Islam y los logros de los eruditos musulmanes en el renacimiento europeo de las matemáticas y la geometría.

Las similitudes estructurales, proporciones y formas de estas dos cúpulas muestran resultados impresionantes. Parece que según las fuentes históricas y los estudios comparativos realizados en este estudio, la cúpula de Soltaniyeh tiene un impacto directo en la construcción de la cúpula de Santa Maria del Fiore, y la teoría propuesta por primera vez por el profesor Sanpaolesi (1972) es concluyente.

Palabras clave: Soltaniyeh, Santa Maria del Fiore, cúpula, Siglo de Oro islámico, Renacimiento

1. Introduction

The dome is one of the earliest human achievements, dating back to the second millennium BC. (Hosseini et al., 2020)

The constructionally simplest, as well as the earliest example of the dome, is the rock-cut tomb. Such tombs have been found in many civilizations, including the Aegean, Egyptian, Greek, Etruscan, Medean, Persian, and Roman. (Melaragno, 1991)

A dome can be thought of as an arch that has been rotated around its central vertical axis. Thus domes, like arches, have a great deal of structural strength when properly built and can span large open spaces without interior supports. (Vasigh & Shiri, 2021)

Persian domes display great diversity both in structure and aesthetics. According to Pope (1976), the dominant element in Persian architecture is the dome which immediately attracts visitors' attention. (Pope & Gluck, 1976)

One of the oldest Persian domes is innumerable samples of oval-shaped domes called 'Chahar-Taqi' (four vaults). Their main features consist of a cubic base of four supporting elements connected by arches and covered by a dome. Creswell (1958) demonstrated that these 'Chahar-Taqi' in Persian pre-Islamic architecture have mainly influenced all later mosques' concepts after the coming of Islam. (Creswell, 1958, p. p. 83) Single-shell domes are older than other types, and they can be thought of as the forerunners of load-bearing domes and the historical origins of dome formation. The

first domes in Iran were oval or ovoid, with various sections and a wide variety of shapes during the Islamic era. This idea has contributed to tracing the origins of the dome form, symbolization, and structural solutions (Memarian, 1988, p. 123).

In general, domes have the following functions and characteristics:

Protect the space from moisture

Constructing a lighter structure and immense cupola

Separation of the weathering surface from the internal shell and thereby substantially giving improved weather protection.

Union of symbolic, aesthetic, and practical consideration

The concrete symbol of civil and spiritual authority

Accentuation of the height

The similarities between the dome of Soltaniyeh in Zanjan and the church of Santa Maria del Fiore in terms of form and the time of construction have created speculation. It is said that these two works are related to each other, and the dome of Soltaniyeh in Zanjan has been used as a model in the construction of Florence Cathedral.

Piero Sanpaolesi¹ has demonstrated that the dome of Santa Maria del Fiore, with its use of double-shelled structure, may have been inspired by that of Soltaniyeh. (Golshan, 2005; Sanpaolesi, 1972) The article examines this theory.

Methods and Materials

1- Research Hypothesis and Questions

The research hypothesis is that “evolution and progress of structural technology is from the east to the west and Soltaniyeh Dome in Iran has been used as a structural pattern for Santa Maria Del Fiore in Italy.”

To examine the above hypothesis, we should answer the following questions:

- 1- Does the Soltaniyeh dome in Iran be considered an inspiration for Santa Maria Del Fiore dome in Italy?
- 2- What evidence can prove the utilization of construction patterns of Soltaniyeh by Santa Maria Del Fiore?

2- Research Methodology

We have used a comparative-descriptive method to answer the research questions and experiment with the hypothesis. This approach has been used in similar architectural researches. (ALAGHMAND, SALEHI, & MOZAFFAR, 2017; Khodabakhshian, 2016; Moghaddasi, Moghaddasi, & Kalantari Khalilabad, 2020) In this process, using reliable library sources, the existing theories in this field are reviewed, and then, by comparing and explaining the differences and similarities, we proceed to answer the research questions. Also, by matching the date of construction of the Iranian and Italian domes to answer the research questions, an attempt has been made to discover the connection between the constructions of two case studies.

1 President of Florence University from April 23, 1969 to July 7, 1971.

3- Comparative Study of the cases

By the comparative-historical study of the domes of Iran and Italy in different historical periods, tables 1 and 2 were obtained.

Thirteen domes from Iran (Iranian plateau) and thirteen domes from Italy (Eastern Roman Empire) were selected, and their construction dates were extracted from reliable sources. Firstly, the selection criteria of these samples are the availability of sufficient information from the date of their construction and, secondly, the fact that they express the style of architecture (historical period). To better understand the information obtained in this section, a timeline is prepared following the order of historical periods. (Table 1) Table 2 shows the domes in terms of geometric form and number of shells in the timeline. These tables help us to study the time of construction of the domes historically and formally.

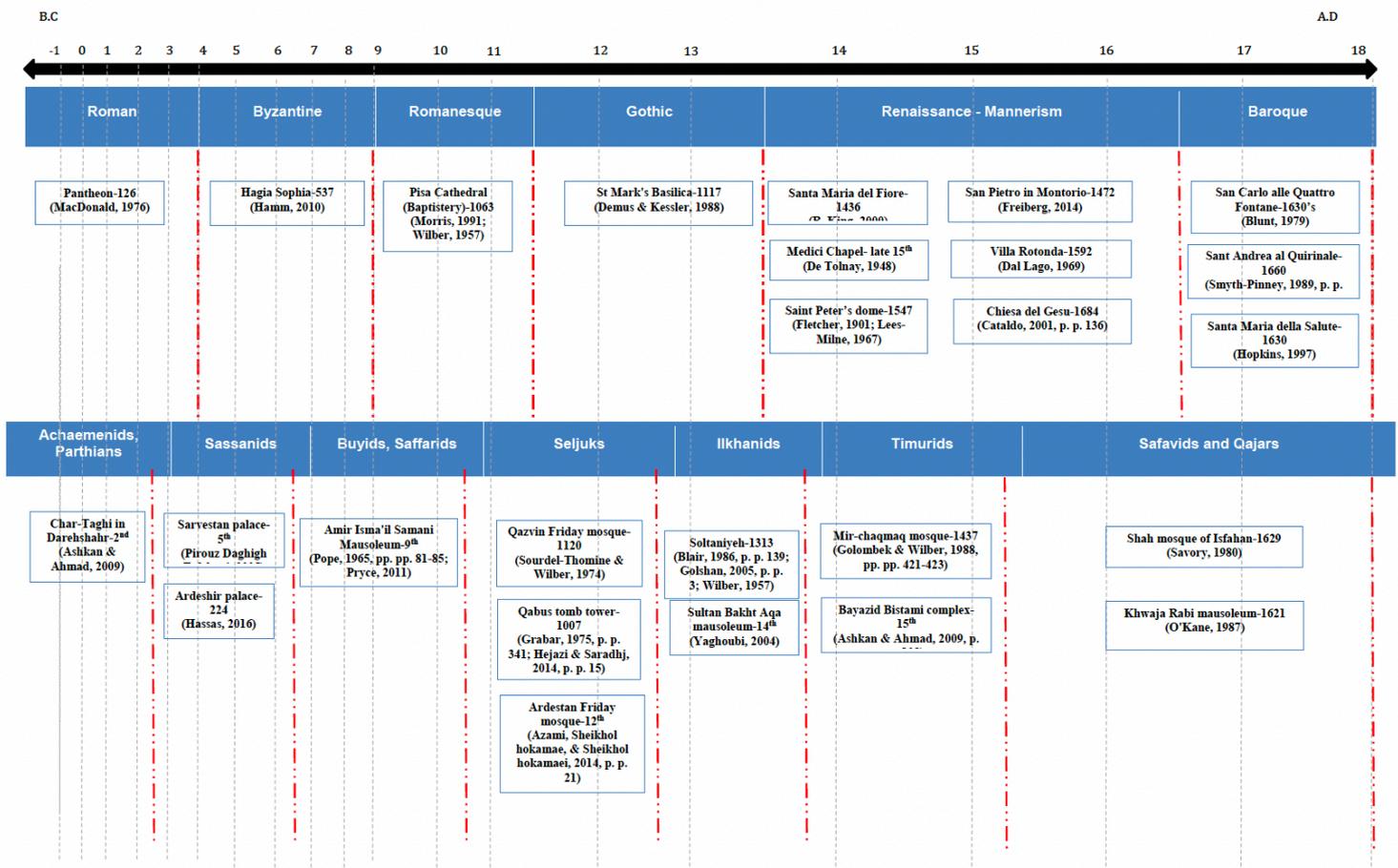
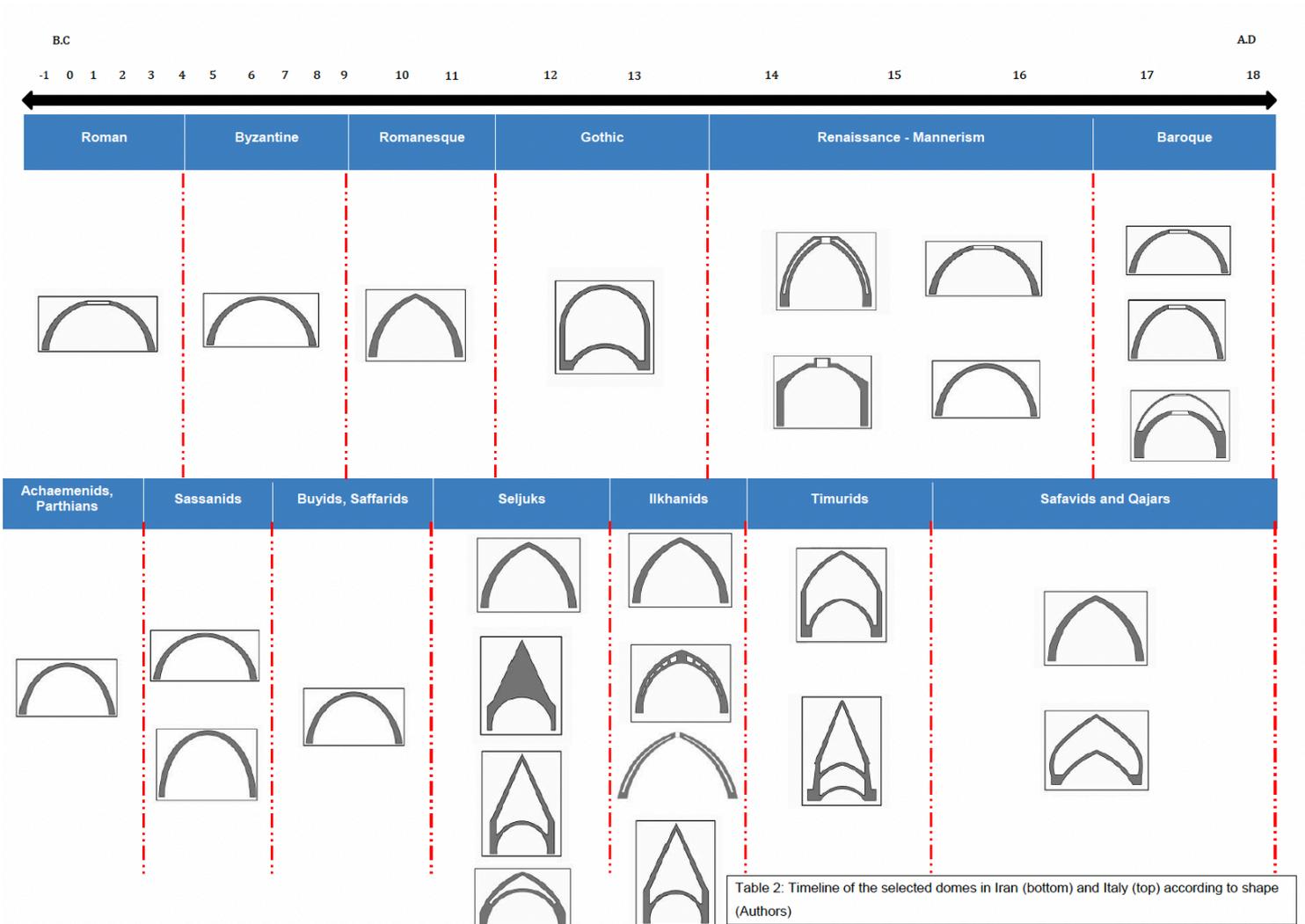


Table 1: Timeline of the selected domes in Iran (bottom) and Italy (top) and date of construction (Authors)

Table 1: Timeline of the selected domes in Iran (bottom) and Italy (top) and date of construction (Ahmad Moghaddasi, Mohammad Hossein Moghaddasi & Seyed Behshid Hosseini)

Table 2: Timeline of the selected domes in Iran (bottom) and Italy (top) according to shape (Ahmad Moghaddasi, Mohammad Hossein Moghaddasi & Seyed Behshid Hosseini)



4 - Available sources

Studying the available sources of reliable scientific centers and the theories of experts who have researched on Soltaniyeh and Santa Maria del Fiore will help to examine the research hypothesis and answer the research questions.

Among the documents for the registration of the Soltaniyeh in World Heritage list, which is available on the website of the WHC, we face this point:

“The cupola of Santa Maria del Fiore known as the Duomo was built on a 13th century cathedral in Florence is the most similar edifice to the Dome of Soltaniyeh.”(Golshan, 2005, p. p. 11)

Piero Sanpaolesi has demonstrated that the dome of Santa Maria del Fiore, with its use of double-shelled structure, may have been inspired by that of Soltaniyeh. (Sanpaolesi, 1972) In addition, the construction system of Soltaniyeh built almost a century before could be known in Italy by the time Brunelleschi began constructing the dome of Santa Maria del Fiore.

Howard Saalman (1980) notes it seems apparent that the double shells concept was firmly determined well before Brunelleschi and Ghiberti came on the scene. He also mentions that:

At all events, the evident success of the dome and its structural system and indirect knowledge of recently-built structures such as the Soltaniyeh Mausoleum. (Saalman, 1980, pp. pp. 80-87)

Soltaniyeh has long been a transit route for European tourists to the east. Historical documents show that various people from Europe visited this city. It is unlikely that these visits have led to the transfer of experiences in construction. From the beginning, travellers heading east always stopped in Soltaniyeh. The mendicant Friar Odoric of Pordenone passed through there about 1320 en route to India and China. More important were the official visitors sent by Christian leaders in Europe anxious for an alliance with the Mongols against the rising threat of the Mamluks in the Mediterranean. Pope John XXII set up an archbishopric at Soltaniyeh in 1318, and archbishops were appointed there until 1425. (Blair, 1986, p. p. 140)

Rashid al-Din presented the work to Oljeytü on 24 Muharram 712/1 June 1312. Thus, our closest contemporary account of the construction of Soltaniyeh, but its bombastic prose yields limited concrete data. (Blair, 1986, p. p. 139)

5 - Influential Scientists of the Islamic golden age

In the 7th and 8th centuries AD, Muslims began to onrush in North Africa and Europe and achieved numerous conquests. (Figure 1) The Islamic Golden Age was a period of cultural, economic, and scientific flourishing in the history of Islam, traditionally dated from the 8th century to the 14th century. (D. A. King, 1983) During this era:

Europe gained access to high-level science, which had its provenance in the works of Islamic scholars.

In the early 15th century, Classical texts, lost to European scholars for centuries, became available.

Some Iranian scholars followed earlier work by Greek mathematicians.

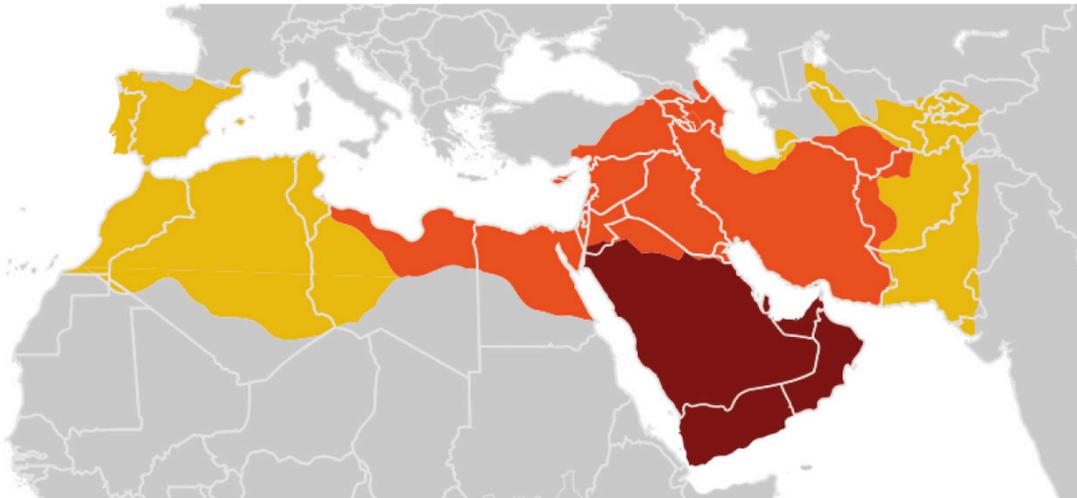


Figure 1. Expansion of Islamic Lands before Islamic Golden Ages 622-750 A.D.(Barraclough & Times Newspapers Limited., 1994)

Many scientists appeared in the golden age of Islam in various fields of science, whose scientific achievements were using for many years. Since the science of construction is profoundly relating to geometry and mathematics, it is clear that the achievements of experts in this field have practically been using in the construction of massive domes and buildings such as Soltaniyeh. It should be noted that the number of scientists in the golden age of Islam, which lasted about 600 years, was numerous, and in this section, we have only selected 5 of them who were outstanding and more influential in mathematics and geometry. (Figure 2)

A review of the achievements of scientists in this field will help to answer the research questions.

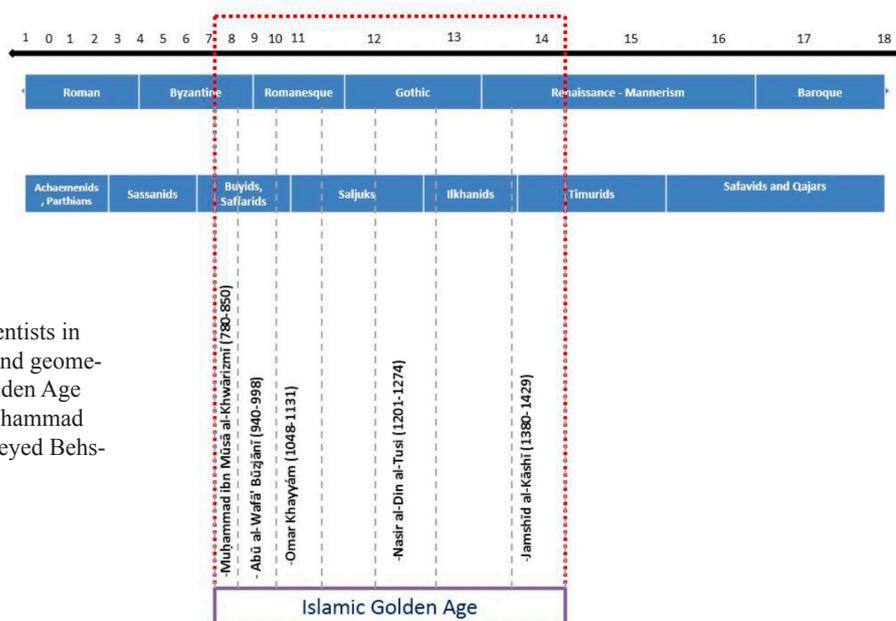


Figure 2. 5 prominent scientists in the field of mathematics and geometry during the Islamic Golden Age (Ahmad Moghaddasi, Mohammad Hossein Moghaddasi & Seyed Behshid Hosseini)

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Investigación

5-1- Khwarazmi

Moḥammad ben Musa Khwarazmi; (c. 780 – c. 850), Arabized as al-Khwarizmi and earlier Latinized as Algorithmi², was a Persian polymath who produced vastly significant works in mathematics, astronomy, and geography. (Berggren, 2016; Corbin, 1998; Pickover, 2009; Saliba, 1998) Around 820 CE, he was appointed as the astronomer and head of the library of the House of Wisdom in Baghdad. (Maher, 1998)

Al-Khwarizmi's approach of solving linear and quadratic equations worked by first reducing the equation to one of six standard forms (where b and c are positive integers) (Rosen, 1937):

- i squares equal roots ($ax^2 = bx$)
- i squares equal number ($ax^2 = c$)
- i roots equal number ($bx = c$)
- i squares and roots equal number ($ax^2 + bx = c$)
- i squares and number equal roots ($ax^2 + c = bx$)
- i roots and number equal squares ($bx + c = ax^2$)

5-2- Buzjani

Abu'l-Wafa' Buzjani (940-998 A.D.) was a great Iranian mathematician and astronomer during the golden age of Islam. (Ben-Menahem, 2009; de Laet, 1994)

He solved the unsolvable problems of classical geometry and researched the principles of geometric drawings, to which no one has yet come up with another solution. Abu al-Wafa is well-known globally for this, and he was the first to conduct detailed studies on the moon. (Hashemipour, 2014)

The science of trigonometry as known today was established by Islamic mathematicians. One of the most important of these was the Persian scholar Abu'l-Wafa' Buzjani (d. 997 or 998), who wrote a work called the Almagest dealing mostly with trigonometry. (de Laet, 1994)

Buzjani established several trigonometric identities such as $\sin(a \pm b)$ in their modern form, where the Ancient Greek mathematicians had expressed the equivalent identities in terms of chords.

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

He also found the law of sines for spherical triangles:

where A, B, C are the sides and a, b, c are the opposing angles

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

The legacy of this document in Latin Europe is still discussed. (Raynaud, 2012)

² Al-Khwarizmi is often considered the founder of algebra, and his name gave rise to the term algorithm

5-3- Khayyam

Omar Khayyam(1048–1131) was a Persian polymath, mathematician, astronomer, philosopher, and poet.(Al-Khalili, 2012) He was born in Neyshabour, in northeastern Persia, and was contemporary with the Seljuks rule near the time of the First Crusade. (Britannica, 2021; Fouchécour & Rosenfeld, 2012)

Khayyam was famous during his times as a mathematician. He wrote the conspicuous Treatise on Demonstration of Problems of Algebra (1070), which laid down the origins of algebra, part of the body of Persian Mathematics that was finally transmitted to Europe. (Akhavan Saber, 2010)

Omar Khayyam created important works on geometry, specifically on the theory of proportions. (Parshall, 1988)

5-4- Al-Tusi

Al-Tusi (1201–1274), better known as Nasir al-Din al-Tusi (in the West), was a Persian polymath, architect, philosopher, physician, scientist, and theologian.(Badakhchani, 1998; Bennison, 2009; Collomb, 2006; de Laet, 1994; Glick, Livesey, & Wallis, 2005; Goldschmidt & Boum, 2018; Joosse, 2004; Mirchandani, 2010; Nasr, 2006)

Nasir al-Din al-Tusi was a well-published author, writing on math, engineering, prose, and mysticism. Additionally, al-Tusi made numerous scientific advancements. In astronomy, al-Tusi created exact tables of planetary motion, an updated planetary model, and critiques of Ptolemaic astronomy. He also invented strides in logic, mathematics but particularly trigonometry, biology, and chemistry. Nasir al-Din al-Tusi left behind a great legacy as well. Some consider Tusi one of the most outstanding scientists of medieval Islam,(Van Brummelen, 2009) since he is often considered the creator of trigonometry as a mathematical discipline in its own right.(Berggren, 2013; Nasr & Leaman, 1996)

Al-Tusi was the first to write a work on trigonometry independently of astronomy. In his Treatise on the Quadrilateral, Al-Tusi gave an extensive exposition of spherical trigonometry, distinct from astronomy.(Katz, 2009)

He was the first to list the six separate cases of a right triangle in spherical trigonometry.

In Al-Tusi's works, trigonometry achieved the status of an independent branch of pure mathematics distinct from astronomy, to which it had been linked for so long. (Asimov & Bosworth, 1998; Badeau & Hayes, 1983)

This subject followed earlier work by Greek mathematicians like Menelaus of Alexandria, who wrote a book on spherical trigonometry called Sphaerica, and the earlier Muslim mathematicians Abu'l-Wafa' Buzjani.(Katz & Imhausen, 2007)

5-5- Al-Kashani

Ghiyas-ud-din Jamshid Kashani (or al-Kashani) (1380–1429) was a Persian astronomer and mathematician during the reign of Tamerlane.(Bosworth, 1990; Selin, 2008)

Much of al-Kashi's work was not brought to Europe, and still, even the extant work remains unpublished in any form.(Bagheri, 1997)

ī Law of cosines

Al-Kashi was the first to provide a precise statement of the law of cosines in a form proper for triangulation.(Pickover, 2009)

ī The Treatise on the Chord and Sine

In algebra and numerical analysis, he developed an iterative method for solving cubic equations, which was not discovered in Europe until centuries later.(Rashed & Armstrong, 1994) In Western Europe, a similar method was later explained by Henry Biggs in his Trigonometria Britannica, published in 1633. (Ypma, 1995)

In order to ascertain $\sin 1^\circ$, al-Kashi discovered the following formula often ascribed to François Viète in the sixteenth century.(Anderson, Katz, & Wilson, 2004)

6- Structures, Proportions and Shapes

6-1- Structural Similarity

By rigorously analyzing the shell of the domes of Soltaniyeh and Santa Maria del Fiore, we observed a major similarity in the dome load distribution model. Both cases transfer the load of the dome shell to the bases of the dome in the lower part through a network of brick beams. (Table 3) There are similar principles for transferring the weight of a dome to the side walls. As shown in Table 3, the main load of the dome is transferred to 8 points at the bases.

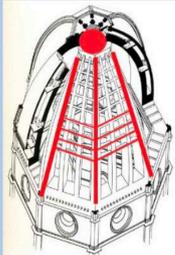
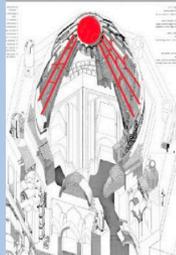
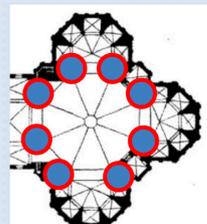
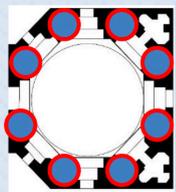
Domes	Santa Maria del Fiore	Soltaniyeh
Item		
Shell load distribution model		
Main points of weight transfer of the dome		

Table 3. Structural similarities in Solatiyeh and Santa Maria del Fiore (Ahmad Moghaddasi, Mohammad Hossein Moghaddasi & Seyed Behshid Hosseini)

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Investigación

6-2- Proportions and Shapes

We measured and examined the proportions in the dome of Soltaniyeh, and Santa Maria del Fiore shows exciting results. Since there are necessary documents of dimensions from both buildings (Golshan, 2005; Sanpaolesi, 1972), we used them to study the height of the domes and the space below them. The diameters of the space under the dome of Santa Maria del Fiore and the dome of Soltaniyeh are 42 meters and 25.5 meters, respectively. Also, the dome’s height from the bases to the lower shell is 39 meters and 25 meters, respectively. The ratio of the diameter of these two domes is 1.6, and the proportion of their height is about 1.5. It seems that the dome of Santa Maria del Fiore was built with a ratio of about 1.5 to the dome of Soltaniyeh, and the Coefficients are almost the same. (Table 4)

There are also many similarities in the plan, the arrangement of the spaces, the shape of the shell and the details of both domes, which are given in Table 5.

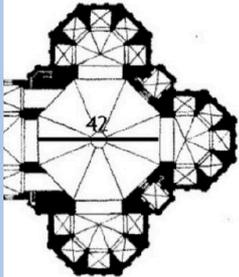
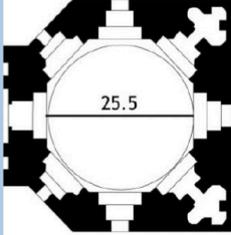
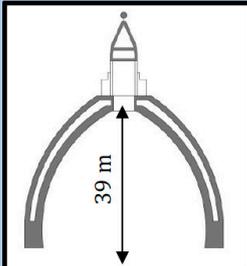
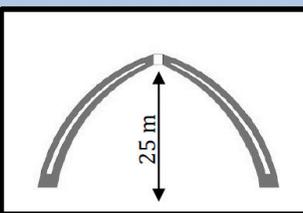
Domes	Santa Maria del Fiore	Soltaniyeh
Item		
Diameter of the space under the dome		
	$\frac{\text{Santa Maria del Fiore}}{\text{Soltaniyeh mausoleum}} = \frac{42}{25.5} \sim 1.6$	
Dome's height		
	$\frac{\text{Santa Maria del Fiore}}{\text{Soltaniyeh mausoleum}} = \frac{39}{25} \sim 1.5$	

Table 4. Proportions in Soltaniyeh and Santa Maria del Fiore (Ahmad Moghaddasi, Mohammad Hossein Moghaddasi & Seyed Behshid Hosseini)

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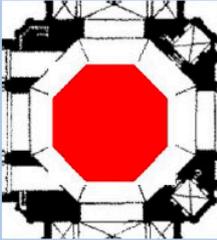
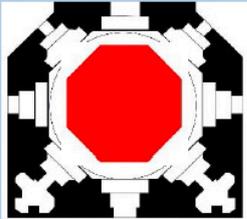
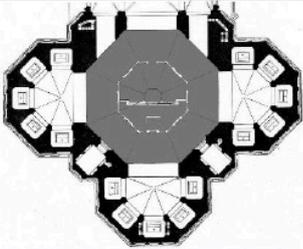
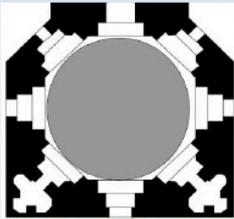
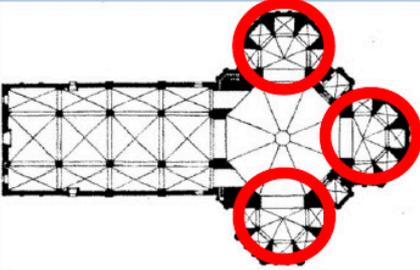
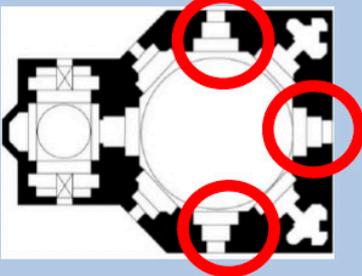
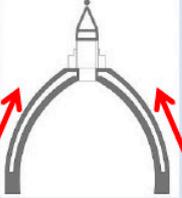
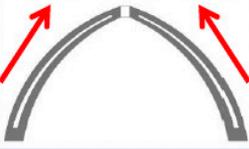
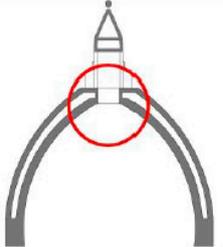
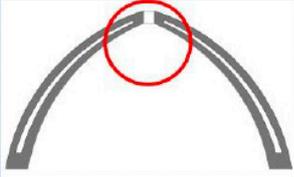
Domes Item	Santa Maria del Fiore	Soltaniyeh
Octagonal plan shape		
Central plan		
Organized Chapels around the center		
Increase in the height of both domes		
Both domes topped with a large flashlight		

Table 5. Formal similarities in Soltaniyeh and Santa Maria del Fiore (Ahmad Moghaddasi, Mohammad Hossein Moghaddasi & Seyed Behshid Hosseini)

Discussion and Conclusion

In answer to the question of research whether the dome of Soltaniyeh can be considered a source of inspiration for the construction of Santa Maria del Fiore, it should be said that according to evidence such as the trip of orientalists to Iran and the Far East in that period, this experience transfer is not far from the mind and could have been done through tourists. Although the oldest surviving engraving of Soltaniyeh dates back to 1537 (Figure 3) (Blair, 1986, p. p. 140; Sanpaolesi, 1972, p. p. 245), there are many documents of various people traveling to Iran during the Soltaniyeh prosperity. Clavijo, a famous Spanish tourist who passed through Iran during the Timurid era, heard and visited the fame of the Dome of Soltaniyeh. (González de Clavijo & López Estrada, 1943)

Also, according to Tables 1 and 2, the time of emergence of double-shell domes in Iran was earlier than in Italy, and their development process was immediate. (Graph 1) Also, the technology of making double-shell domes with wide spans in Iran before the construction of the Cathedral of Florence was fully developed.

Regarding the second question of the research, what is the evidence that confirms the use of the construction patterns of the Soltaniyeh dome in Santa Maria del Fiore? First, the influence of scientists of the Islamic Golden Age on the European Renaissance. The achievements of Iranian mathematicians who have been practically used in architecture and building-related sciences are indisputable. (Berggren, 2016; Glick et al., 2005) Second, structural similarities, close proportions, similar spatial arrangement, and the domes' shape prove this claim. (Tables 3, 4, 5) Third, these two buildings' construction techniques and materials have been studied and compared more closely by Sanpaolesi. (Sanpaolesi, 1972, pp. pp. 250-258)

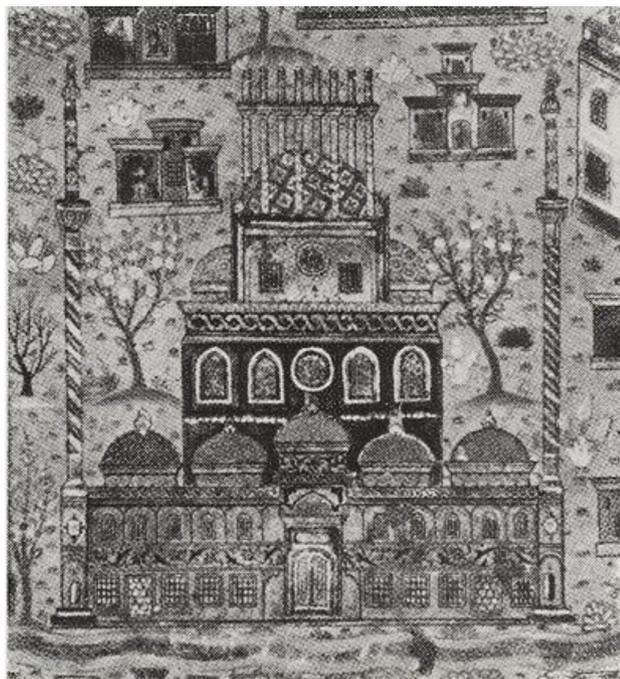
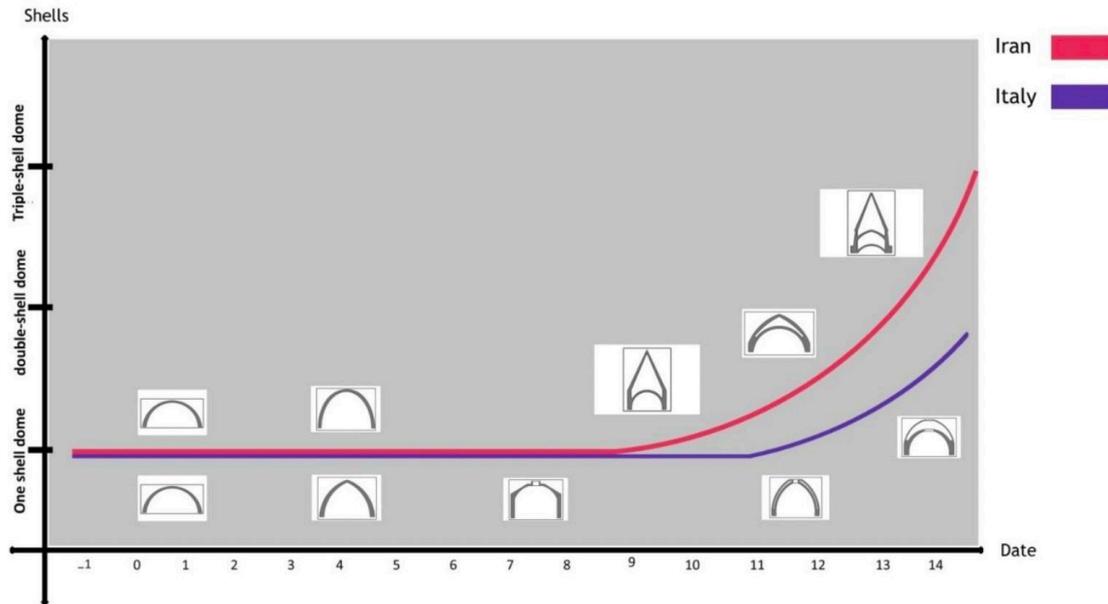


Figure 3. The oldest drawing of Soltaniyeh Dome, 1537 A.D. (Blair, 1986)



Graph 1. Development of the Shells in Iranian and Italian Domes during ages, A Schematic Graph (Ahmad Moghaddasi, Mohammad Hossein Moghaddasi & Seyed Behshid Hosseini)

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Investigación

In our studies, we confirmed that the dome of Soltaniyeh was constructed before the dome of Santa Maria del Fiore, and there was no other significant example of double shells dome in Italy before that time. Moreover, we derived that the historical development of dome construction in Iran was more technologically advanced in those eras than in Italy. Also, in our inquiries regarding the structural aspect, Available sources, Proportion, Shape, and Material and influential Scientists of the Islamic golden age, it became more apparent that both cases are connected firmly in diverse ways.

Therefore, the research, according to 4 separate ways of investigating the hypothesis (depicting the information in charts, investigating the scientific situation of Iran, and analyzing the shape and proportion), indicated that the hypothesis is accurate and there is a strong relationship between the construction of the dome of Santa Maria del Fiore and the Dome of Soltaniyeh.

It means that the Renaissance in Europe, even in art and architecture, was strongly influenced by the achievements of the architects of Islamic countries. Nor should we neglect Muslim influence in southern Europe, which lasted more than 750 years until the end of the fifteenth century.

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